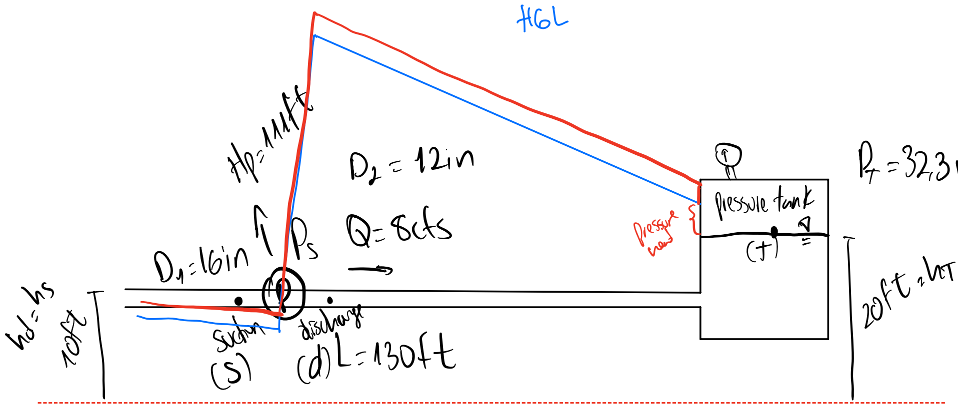
1) (Prob 4.2.11)

A pump is needed to convey 8cfs of water through a 130-ft-long horizontal, ductile iron pipeline to a pressurized tank. The pipeline is installed at an elevation of 10ft, the water surface elevation in the receiving tank is 20ft, and the pressure at the top of the tank is 32.3psi. The pipe has a 16-in-diameter on the suction side of the pump and a 12-in-diamter on the discharge side of the pump. If the pump delivers 111ft of head, determine the pressure head on the discharge side of the pump (in psi) and whether cavitation is a concern on the suction side of the pump? Sketch the EGL and HGL of the system.



$P_T = 32{,}3\ psi = 32{,}3\ lb/in^2$

$\gamma = 62{,}3\ lb/ft^3$

$H_d = H_T + h_L \rightarrow$ to know the P_d

$$h_d + \frac{P_d}{\gamma} + \frac{V_d^2}{2g} = h_T + \frac{P_T}{\gamma} + \cancel{\frac{V_T^2}{2g}}^{\,0} + f\frac{L}{D}\frac{V^2}{2g} + K_d\frac{V^2}{2g}$$

$$10 + \frac{P_d}{\gamma} + \frac{V_d^2}{2\cdot 32{,}2} = 20 + \frac{32{,}3\ \frac{lb}{in^2}}{62{,}3\ \frac{lb}{ft^3}}\cdot\left(\frac{12\,in}{1\,ft}\right)^2 + f\cdot\frac{130\,ft}{12in\cdot\frac{1ft}{12in}}\cdot\frac{V^2}{2\cdot 32{,}2} + 1\cdot\frac{V^2}{2\cdot 32{,}2}$$

$$10 + \frac{P_d}{\gamma} + 0{,}0155V_d^2 = 20 + 74{,}66 + 2{,}019\, f\cdot V^2 + 0{,}0155\,V^2$$

$$\frac{P_d}{\gamma} = 84{,}66 + 2{,}019\, fV^2 + \cancel{0{,}0155V^2} - \cancel{0{,}0155V_d^2}$$; $V_d = V$ in the pipe

$$\frac{P_d}{\gamma} = 84{,}66 + 2{,}019\, fV^2 \quad (eq\ 1)$$

$$V_d = ?? \Rightarrow V_d = \frac{Q}{A} = \frac{8\,ft^3/s}{\frac{\pi}{4}\left(12in\cdot\frac{1ft}{12in}\right)^2} = 10{,}19\ ft/s$$

$$f = ??\quad \frac{e}{D} = \frac{0{,}0004ft}{12in\cdot\frac{1ft}{12in}} \Rightarrow \frac{e}{D} = 0{,}0004\ ,\quad N_R = \frac{DV}{\nu} = \frac{12in\cdot\frac{1ft}{12in}\cdot 10{,}2}{1{,}08\times10^{-5}} \Rightarrow N_R = 9{,}44\times10^5$$

Using Moody diagram $f = 0{,}017$

replace V_d and f

$$\frac{P_d}{\gamma} = 84,66 + 2,014 \cdot 0,017 \cdot (10,19)^2$$

$$\frac{P_d}{\gamma} = 88,22\ ft \Rightarrow P_d = 88,22\ \cancel{ft} \cdot 62.3\ \frac{lb}{\cancel{ft^3}} \left(\frac{1ft}{12in}\right)^2 \Rightarrow P_d = 38,2\ psi$$

↳

$$H_s + H_p = H_d$$

$$\cancel{h_s} + \frac{V_s^2}{2g} + \frac{P_s}{\gamma} + H_p = \cancel{h_d} + \frac{V_d^2}{2g} + \frac{P_d}{\gamma} \quad , \quad h_s = 10ft, \ h_d = 10ft$$

$$\frac{P_s}{\gamma} = \frac{V_d^2}{2g} + \frac{P_d}{\gamma} - \frac{V_s^2}{2g} - H_p \qquad (eq. 2)$$

$V_s = ??$

$$V_s = \frac{Q}{A} = \frac{8cfs}{\frac{\pi}{4}\left(16in \cdot \frac{1ft}{12in}\right)^2} \Rightarrow V_s = 5,73\ ft/s$$

$$\frac{P_s}{\gamma} = \frac{10,19^2}{2 \cdot 32,2} + 88,22ft - \frac{5,73^2}{2 \cdot 32,2} - 111\ ft$$

$$\frac{P_s}{\gamma} = -21,26ft$$ ✓ pag 99 last sentence

negative pressure $> -7m$

$> -23 ft$

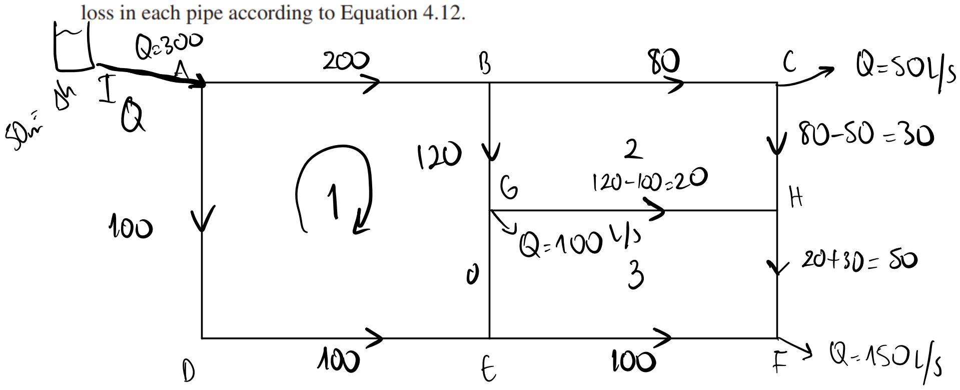
no cavitation problems

2)

Example 4.8

A water-supply distribution system for an industrial park is schematically shown in Figure 4.9 (a). The demands on the system are currently at junctions *C*, *G*, and *F* with flow rates given in liters per second. Water enters the system at junction *A* from a water storage tank on a hill. The water surface elevation in the tank is 50 m above the elevation of point A in the industrial park. All the junctions have the same elevation as point A. All pipes are aged ductile iron ($e = 0.26$ mm) with lengths and diameters provided in the table below. Calculate the flow rate in each pipe. Also determine if the pressure at junction *F* will be high enough to satisfy the customer there. The required pressure is 185 kPa.

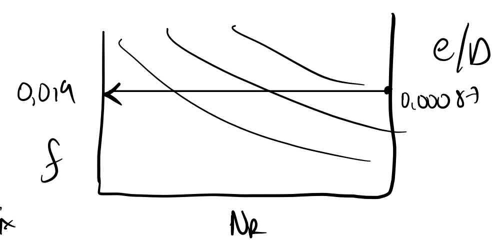
A table of pipe and system geometry is a convenient way to organize the available information and make some preliminary calculations. The table below has been set up for that purpose. The first column identifies all of the pipes in the network. Column 2 contains flow rates for each pipe, which were estimated to initiate the Hardy–Cross algorithm. These estimated flow rates and directions are shown on the system schematic in Figure 4.9(a). Note that mass balance was maintained at each junction. Friction factors (column 6) are found assuming complete turbulence and read from the Moody diagram using *e*/*D* or, alternatively, from Equation 3.23. The "*K*" coefficient (column 7) is used later in the procedure to obtain the head loss in each pipe according to Equation 4.12.



e = 0,26 mm

$Q_A = 50+100+150 \Rightarrow Q_A = 300\,l/s$

Pipe	Flow (m³/s)	Length (m)	Diameter (m)	*e*/*D*	*f*	*K* (s²/m⁵)
AB	200 l/s = 0.20	300	0.30	0.00087	0.019	194
AD	0.10	250	0.25	0.00104	0.020	423
BC	0.08	350	0.20	0.00130	0.021	1,900
BG	0.12	125	0.20	0.00130	0.021	678
GH	0.02	350	0.20	0.00130	0.021	1,900
CH	0.03	125	0.20	0.00130	0.021	678
DE	0.10	300	0.20	0.00130	0.021	1,630
GE	0.00	125	0.15	0.00173	0.022	2,990
EF	0.10	350	0.20	0.00130	0.021	1,900
HF	0.05	125	0.15	0.00173	0.022	2,990



$$h_f = f\,\frac{L}{D}\,\frac{\overset{v=Q/A}{V^2}}{2g} = \underbrace{\left[f\,\frac{L}{D}\,\frac{1}{2gA^2}\right]}_{K} Q^2$$

Q^2

hardy-Cross method uses the following eq. to estimate the ΔQ

$$\Delta Q = \frac{\sum h_{fc} - \sum h_{fcc}}{\beta\left(\sum \frac{h_{fc}}{Q_c} + \sum \frac{h_{fcc}}{Q_{cc}}\right)}$$

$\beta = 1{,}85$ hazen-williams

$\beta = 2$ Darcy-Weisbach

→ Loop 1.

Pipe		Q (m³/s)	K	$h_f = KQ^2$	h_f/Q	Qnew
AB	c	0,2	194	7,76	38,8	0,2+|-0,005| = 0,205
BG	c	0,12	678	9,76	81,3	0,12+ 〃 = 0,125
GE	c	0	2990	0	0	0 + 〃 = 0,005
AD	cc	-0,1	423	4,23	42,3	-0,1+ 〃 = -0,095
DE	cc	-0,1	1630	16,3	163,0	-0,1 + 〃 = -0,095

$$\Delta Q = \frac{\sum h_{fc} - \sum h_{fcc}}{2\cdot\left[\sum \frac{h_{fc}}{Q_c} + \sum \frac{h_{fcc}}{Q_{cc}}\right]} = \frac{(7{,}76+9{,}76)-(4{,}23+16{,}3)}{2\left[(38{,}8+81{,}3)+(42{,}3+163)\right]}$$

$\Delta Q = -0{,}005\ m^3/s$

$\Delta Q > 0 \Rightarrow Q_{new} = Q - |\Delta Q|$

$\Delta Q < 0 \Rightarrow Q_{new} = Q + |\Delta Q|$

→ Loop 2

Pipe		Q	K	h_f	h_f/Q	Qnew
BC	c	0,08	1900	12,2	152,5	0,08 − |0,002| = 0,078
CH	c	0,03	678	0,61	20,3	0,03− 〃 = 0,028
✓ BG	cc	-0,125	678	10,6	84,8	-0,125 − 〃 = -0,127
GH	cc	-0,02	1900	0,76	38	-0,02 − 〃 = -0,022

$$\Delta Q = \frac{[12{,}2+0{,}61]-[10{,}6+0{,}76]}{2[(152{,}5+20{,}3)+(84{,}8+38)]} \Rightarrow \Delta Q = 0{,}002\ m^3/s$$

→ Loop 3

Pipe		Q	K	h_f	h_f/Q	Qnew	
✓ GH	c	0,022	1900	0,92	41,8	0,022+|-0,013| = 0,035	c
HF	c	0,05	2990	7,48	149,6	0,05+ 〃 = 0,063	c
✓ GE	cc	-0,005	2990	0,07	14	-0,005+|-0,013| = 0,008	c
EF	cc	-0,1	1900	19	190	-0,1 + 〃 = -0,087	cc

$\Delta Q = -0{,}013\ m^3/s$

$|\Delta Q| < 0{,}005$

→ Loop 1.

Pipe		$Q\,(m^3/s)$	K	h_f	h_f/Q	Q_{new}
AB	c	0,205	194	8,15	39,8	0,205
BG	c	0,127	678	10,9	85,8	0,127
Direction changed FG	cc	0,008	2990	0,19	23,8	0,008
AD	cc	0,095	423	3,82	40,2	0,095
DE	cc	0,095	1630	14,7	154,7	0,095

$\Delta Q = 0{,}000494 \approx 0{,}000$

→ Loop 2

Pipe		Q	K	h_f	h_f/Q	Q_{new}
BC	c	0,078	1900	11,6	148,7	0,078 + (0,002) = 0,08
CH	c	0,028	678	0,53	18,4	0,028 + 〃 = 0,03
✓ BG	cc	-0,127	678	10,9	85,8	-0,127 + 〃 = -0,125
GH	cc	-0,035	1900	2,33	66,6	-0,035 + 〃 = -0,033

$\Delta Q = -0{,}002$

→ loop 3

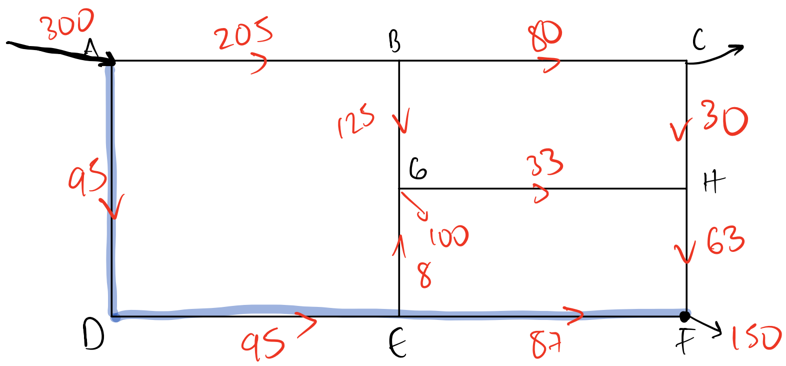
Pipe		Q	K	h_f	h_f/Q	Q_{new}
✓ GH	c	0,033	1900	2,07	62,7	0,033
HF	c	0,063	2990	11,9	188,9	0,063
✓ GE	cc	-0,008	2990	0,19	23,8	0,008
EF	cc	-0,087	1900	14,4	165,5	-0,087

$\Delta Q = -0{,}000272 \approx 0{,}000$

Pipe	Q
AB	205
AD	95
BC	80
BG	125
GH	33
CH	30
DE	95
EG	8
EF	87
HF	63

Let's see if $P_F \geq 185\,kPa$

$$P = \gamma \cdot h = 9790\,N/m^3 \cdot 50m \Rightarrow P = 489{,}5\,kPa$$



$$P_F = P_A - \Delta P_{AD} - \Delta P_{DE} - \Delta P_{EF} \qquad \Delta P = h_f \cdot \gamma$$

$$= 489{,}5 - 9{,}79 \cdot 3{,}82 - 9{,}79 \cdot 14{,}7 - 9{,}79 \cdot 14.4$$

$$P_F = 167{,}21\,kPa < 185\,kPa$$

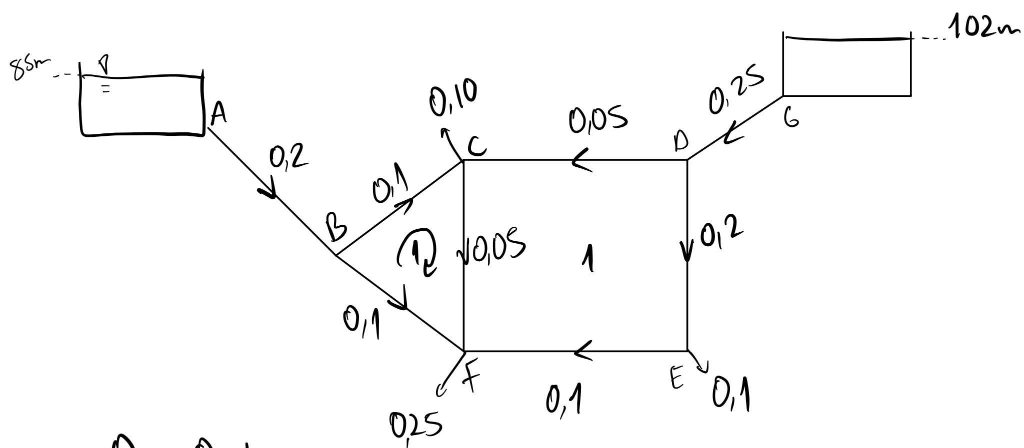
increase elevation of the tank $\Rightarrow >P_A$
or modify pipe network

3)

Example 4.9

Consider the pipe network shown in Figure 4.10(a), which contains two reservoir sources. Suppose $H_A = 85$ m, $H_G = 102$ m, $Q_c = 0.10$ m^3/s, $Q_F = 0.25$ m^3/s, and $Q_E = 0.10$ m^3/s. The pipe and junction characteristics are tabulated below. Also tabulated are the initial estimates of the flow rates in all the pipes (cast iron; $e = 0.26$ mm). The flow directions are shown in Figure 4.10(a). Determine the discharge in each pipe and the pressure head at each junction.

Pipe	Length (m)	Diameter (m)	e/D	f	K (s^2/m^5)	Q (m^3/s)	Junction	Elev. (m)
AB	300	0.30	0.00087	0.019	194	0.200	A	48
BC	350	0.20	0.00130	0.021	1,900	0.100	B	46
BF	350	0.20	0.00130	0.021	1,900	0.100	C	43
CF	125	0.20	0.00130	0.021	678	0.050	D	48
DC	300	0.20	0.00130	0.021	1,630	0.050	E	44
EF	300	0.20	0.00130	0.021	1,630	0.100	F	48
DE	125	0.20	0.00130	0.021	678	0.200	G	60
GD	250	0.25	0.00104	0.020	423	0.250		



$Q_{in} = Q_{out}$

$Q_{in} = 0{,}1 + 0{,}25 + 0{,}1$

$Q_{in} = 0{,}45$

Loop 1

Pipe		Q	K	h_f	h_f/Q	Q_{new}
BC	c	0,1	1900	19	190	$0{,}1 - \lvert 0{,}002 \rvert = 0{,}098$
CF	c	0,05	678	1,7	33,9	$0{,}05 - \lvert 0{,}002 \rvert = 0{,}048$
BF	cc	-0,1	1900	-19	190	$0{,}1 - 0{,}002 = 0{,}098$

$$\Delta Q = \frac{\sum h_{fc} - \sum h_{fcc}}{2\left[\sum \frac{h_{fc}}{Q_c} + \sum \frac{h_{fcc}}{Q_{cc}}\right]} = \frac{19 + 1{,}7 - 19}{2\left[190 + 33{,}9 + 190\right]}$$

$\Delta Q > 0 \Rightarrow Q_{new} = Q - |\Delta Q|$

$\Delta Q < 0 \Rightarrow Q_{new} = Q + |\Delta Q|$

$\Delta Q = 0{,}002$ m^3/s

loop 2

Pipe		Q	K	h_f	h_f/Q	Q_{new}
DE	c	0,2	678	27,12	135,6	0,2 − |0,046| = 0,154
EF	c	0,1	1630	16,3	163	0,1 − |0,046| = 0,054
DC	cc	−0,05	1630	4,08	81,5	−0,05 − |0,046| = 0,096
✓ CF	cc	−0,048	678	1,56	32,54	−0,048 − 0,046 = 0,094

$\Delta Q = 0{,}046 \ m^3/s$

Inflow path	Pipe		Q	K	h_f	h_f/Q	Q_{new}
ABCDG	AB	c	0,2	194	7,76	38,8	0,2 − |0,002| = 0,198
	✓ BC	c	0,098	1900	18,25	186,20	0,098 − |0,002| = 0,096
	✓ DC	cc	−0,096	1630	15,02	156,48	−0,096 − " = −0,098
	GD	cc	−0,25	423	26,44	105,75	−0,25 − " = −0,252

$$\Delta Q = \frac{\sum h_{fp} - \sum h_{fcp} + H_G - H_A}{2\left[\sum \frac{h_{fp}}{Q_p} + \sum \frac{h_{fcp}}{Q_{cp}}\right]} = \frac{(7{,}76 + 18{,}25) - (15{,}02 + 26{,}44) + 102 - 85}{2\left[(38{,}8 + 186{,}2) + (156{,}48 + 105{,}75)\right]}$$

$\Delta Q = 0{,}002 \ m^3/s$

$|\Delta Q| < 0{,}005$

- Loop 1

Pipe		Q	K	h_f	h_f/Q	Q_{new}
BC	c	0,096	1900	17,51	182,4	0,096 − |0,004| = 0,096
CF	c	0,094	678	5,99	63,73	0,094 − " = 0,09
BF	cc	−0,102	1900	19,77	193,80	−0,102 − " = −0,106

$\Delta Q = 0{,}004 \ m^3/s$ ✓

→ loop 2

Pipe		Q	K	h_f	h_f/Q	Q_{new}
DE	c	0,154	678	16,08	104,41	0,154
EF	c	0,054	1630	4,75	88,02	0,054
DC	cc	−0,098	1630	15,65	159,74	−0,098
✓ CF	cc	−0,090	678	5,49	61,02	−0,09

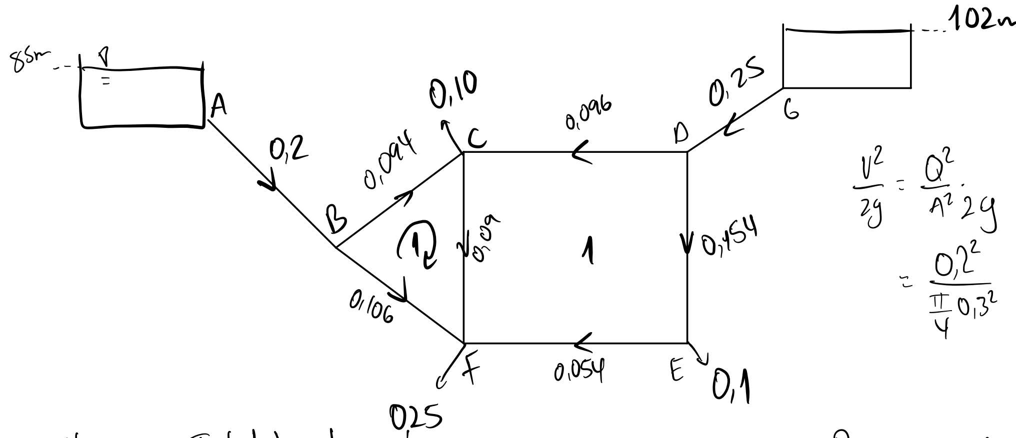
$\Delta Q = -0.00037 \simeq 0{,}000$

Inflow path	Pipe	Q	K	h_f	h_f/Q	Q_{new}
ABCDG	AB	≈ 0,198	194	7,61	38,41	0,198 + |0,002| = 0,2
	✓ BC	≈ 0,092	1900	16,08	174,8	0,092 + |0,002| = 0,094
	✓ DC	≈ -0,098	1630	15,65	159,74	-0,098 + " = 0,096
	GD	≈ -0,252	423	26,86	106,6	-0,252 + " = 0,25

$\Delta Q = -0{,}002\ m^3/s$

stop

Pipe	Q (m³/s)	h_f (m)	Junction	Elevation (m)	Total Head (m)	Pressure Head (m)
AB	0.200	7.76	A	48.00	85.00	37.00
BC	0.094	16.79	B	46.00	77.24	31.24
BF	0.106	21.34	C	43.00	60.45	17.45
CF	0.090	5.49	D	48.00	75.56	27.56
DC	0.096	15.02	E	44.00	59.48	15.48
EF	0.054	4.75	F	48.00	55.90	7.90
DE	0.154	16.08	G	60.00	102.00	42.00
GD	0.250	26.44				



$$\frac{V^2}{2g} = \frac{Q^2}{A^2 2g} = \frac{0{,}2^2}{\frac{\pi}{4}0{,}3^2} \cdot \frac{1}{2 \cdot 9{,}81} = 0{,}408$$

	Elev	Total head		Pressure head
A	48	H_A	85 = 85	85 − 48 = 37
B	46	$H_A - h_{fAB}$	85 − 7,76 = 77,24	31,24
C	43	$H_A - h_{fAB} - h_{fBC}$	85 − 7,76 − 16,79 = 60,45	17,45
D	48	$H_G - h_{fGD}$	102 − 26,44 = 75,56	27,56
E	44	$H_G - h_{fGD} - h_{fDE}$	102 − 26,44 − 16,08 = 59,48	15,48
F	48	$H_A - h_{fAB} - h_{fBF}$	85 − 7,76 − 21,34 = 55,9	7,9
G	60	H_G	102	42