Experiment 3

Verification of Bernoulli's Theorem

Objective:

- The purpose of this experiment is to illustrate Bernoulli's Theorem by demonstrating the relationship between pressure head and kinetic energy head for a conduit of varying cross-section.

Pre-Lab Setup:

1. Set up the Bernoulli apparatus on the working surface and level it.
2. Connect the supply hose to the inlet stub and tighten the hose.
3. If not already open, open the outlet valve on the Apparatus.

Tapping Position	Manometer Legend	Diameter (mm)
A	h_1	25.0
B	h_2	13.9
C	h_3	11.8
D	h_4	10.7
E	h_5	10.0
F	h_6	25.0

Note: The assumed datum position is at tapping A associated with h1

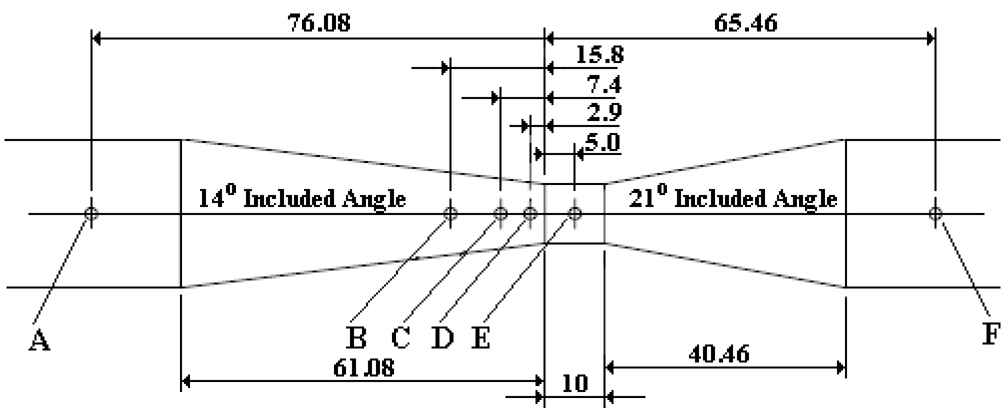

Figure 3-1 Bernoulli Apparatus

Theory:

Bernoulli's equation is derived by integrating the equations of fluid motion. Assumptions used to obtain the simplified version of the equation are that the fluid is inviscid and incompressible and that the flow is steady. Bernoulli's equation is a mathematical statement of the work-energy principle which directly corresponds to the equations of motion. This principle states that the work done on a particle is equal to the change in kinetic energy of the particle. Along a streamline,

$$\frac{p}{\gamma}+\frac{v^2}{2g}+z=const. \qquad (3\text{-}1)$$

Conservation of Mass:

For a given cross-sectional area the product of the velocity and density is proportional to the mass flow rate.

$$M=\rho Q=\rho v A_n \qquad (3\text{-}2)$$

$$v=\frac{M}{\rho A_n}=\frac{(mass\,/\,time)}{(mass\,/\,vol)\,A_n}=\frac{(vol\,/\,time)}{A_n}=\frac{Q}{A_n} \qquad (3\text{-}3)$$

$Q=vA_n$ (continuity equation for incompressible fluid)

where, *M* = mass flow rate,

Q = volumetric flow rate,

v = average velocity,

A_n = area normal to the direction of flow, and

ρ = mass density.

Between any two points in the flow, Inflow = Outflow. Therefore,

$$M_{in}=M_{out} \qquad (3\text{-}4)$$

$$\rho_1 v_1 A_1=\rho_2 v_2 A_2 \qquad (3\text{-}5)$$

which for an incompressible fluid becomes,

$$v_1 A_1=v_2 A_2=Q \qquad (3\text{-}6)$$

If the cross-sectional area decreases, the velocity must increase to satisfy continuity. Applying Bernoulli's equation to a flow where there is no change in elevation (z = constant), a decrease in velocity must be accompanied by an increase in pressure and vice versa. Bernoulli's equation expresses the conservation of energy and that the work done on the fluid shows up as a change in kinetic and/or potential energy.

The total head h_t is the addition of the static head h and the dynamic, or velocity, head h_d. In other words

$$h_t = h + h_d = \frac{p}{\gamma} + \frac{v^2}{2}$$

Procedure:

1. Readings should be taken at 3 different flow rates with long taper upstream and the total head tube fully retracted from the test-section.

2. Close the main control valve and start the pump.

3. To bleed air from the flexible connections and the manometer tubes connect a length of small bore tubing from the air bleed connection to the overflow cut-out in the side of the volumetric tank.

   Gradually close the close outlet flow control valve to increase the pressure in the test section, then open the air bleed screw until all air bubbles have been flushed from the system. When all the manometer tubes are completely full of water, close the air bleed screw then open the outlet flow control valve to give a small flow through the test section. Connect the hand air pump to the air bleed connection, open the air bleed screw then apply pressure using the hand pump until the indicated levels on the manometer are located at around the 180 or 170mm marking; this provides for a good initial level horizon. Close the air bleed valve. The manometer equipment is ready to use.

4. Take the first set of readings at the maximum flow rate possible (with all manometers reading on the backboard). The maximum volume flow rate will be determined by the need to have the maximum (h1) and minimum (h5) manometer readings both with the range of the scale. You may need to adjust the pump flow rate (on the bench) and the outflow valve (on the Apparatus) simultaneously to have this 'optimum' spread across the backboard.

5. Record the associated flow rate with a timed volume collection. This can be done on the hydraulic bench reading the water levels and timing how much time it takes to add 10L or 20L and so on.

6. Take the total pressure head distribution by traversing the total head tube along the length of the test section. Record the total head reading on manometer h_8 with the tip of the probe adjacent to each tapping. The datum line is the side hole pressure tapping A associated with manometer h_1.

7. Increase the back pressure in the channel (thus reducing the volume flow rate) by throttling the outflow valve to give a maximum head difference of about 50 mm on the manometer.

8. Again, take the readings of the levels in manometers $h_1 - h_5$. Then repeat the procedure to record the total head at each tapping by traversing the total head tube along the test section.

9. Increase the back pressure one more time for the third flow rate reading by throttling the outflow valve to give the $h_1 - h_5$ difference approximately half way between that obtained in the above two settings.

10. As before, take manometer readings $h_1 - h_5$ and use the total head tube to traverse the test section for the total head readings at each tapping (tube h_8).

11. Switch off the pump and close the main valve.

Results/Questions:

1. Using your measured discharge rate, calculate the velocities at each of the tappings.

2. Calculate the total head, h_t, at each of the cross sections by adding your recorded static head and the calculated velocity head.

3. Plot the total head, h_t, as a function of distance, x, where x = 0 at tapping “A” and x = 14.54 cm at the outlet (tapping F).

4. What is the head loss between the inlet and the throat?

5. What is the head loss between the throat and the outlet?

6. Plot the calculated total head from above and also the measured total head (tube h_8) into a single graph. Do they differ? If so, why do you think this is the case? Discuss observed discrepancies.

7. Calculate the degree of pressure recovery. What does this indicate about the energy of the fluid as it passes through contractions and expansions?

Data:

Record your total head readings from tube h_8 in the last column.

Flow Rate 1:

Volume Collected V (m³)	Time to Collect t (sec)	Flow Rate Q_v (m³/sec)		Distance into Duct (m)	Area of Duct A (m²)	Static Head h (m)	Velocity v (m/s)	Dynamic Head h_d (m)	Total Head h_t (m)
			h_1	0.00	490.9×10^{-6}				
			h_2	0.0603	151.7×10^{-6}				
			h_3	0.0687	109.4×10^{-6}				
Average flow rate			h_4	0.0732	89.9×10^{-6}				
			h_5	0.0811	78.5×10^{-6}				
			h_6	0.1415	490.9×10^{-6}				

Flow Rate 2:

Volume Collected V (m³)	Time to Collect t (sec)	Flow Rate Q_v (m³/sec)		Distance into Duct (m)	Area of Duct A (m²)	Static Head h (m)	Velocity v (m/s)	Dynamic Head h_d (m)	Total Head h_t (m)
			h_1	0.00	490.9×10^{-6}				
			h_2	0.0603	151.7×10^{-6}				
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Average flow rate			h_4	0.0732	89.9×10^{-6}				
			h_5	0.0811	78.5×10^{-6}				
			h_6	0.1415	490.9×10^{-6}				

Flow Rate 3:

Volume Collected V (m³)	Time to Collect t (sec)	Flow Rate Q_v (m³/sec)		Distance into Duct (m)	Area of Duct A (m²)	Static Head h (m)	Velocity v (m/s)	Dynamic Head h_d (m)	Total Head h_t (m)
			h_1	0.00	490.9×10^{-6}				
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